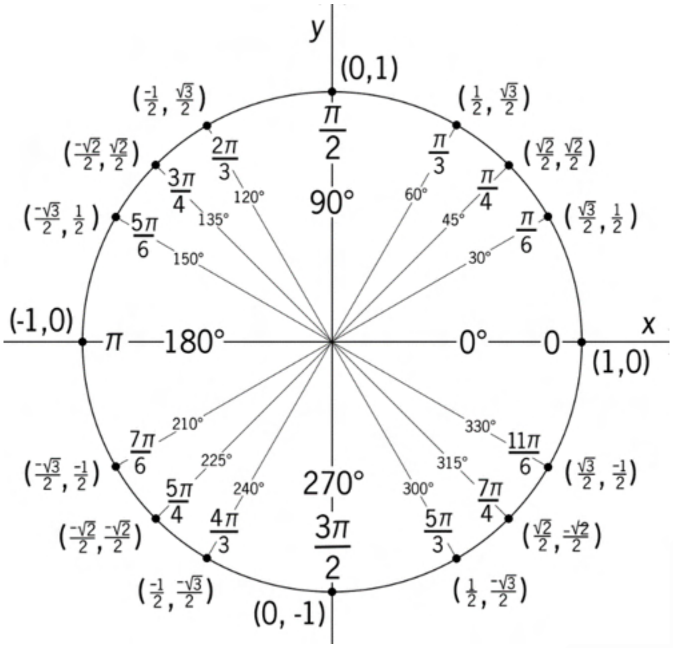
Crafton Hills College Tutoring Center

Trigonometry Formulas and Properties



Reciprocal Identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Tangent and Cotangent Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas:

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Cofunction Formulas:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Product to Sum Formulas:

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Sum and Difference Formulas:

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Periodic Formulas:

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Half-Angle Formulas:

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

Double Angle Formulas:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

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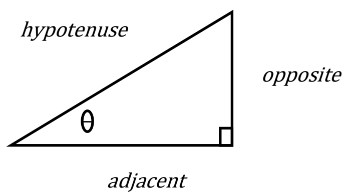
Trigonometric Functions:

Right Triangle:

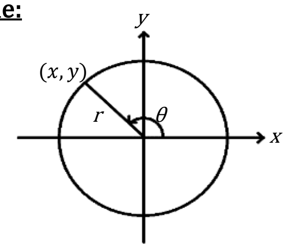


$\sin\theta = \dfrac{\text{opposite}}{\text{hypotnuse}}$ $\csc\theta = \dfrac{\text{hypotnuse}}{\text{opposite}}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotnuse}}$ $\sec\theta = \dfrac{\text{hypotnuse}}{\text{adjacent}}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}}$

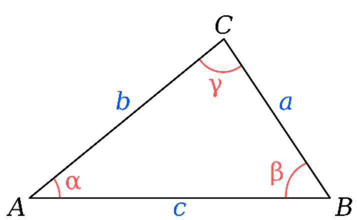
Unit Circle:



$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$



Law of Sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Law of Tangents:

$\dfrac{a-b}{a+b} = \dfrac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$

$\dfrac{b-c}{b+c} = \dfrac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$

$\dfrac{a-c}{a+c} = \dfrac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$

Inverse Trigonometric Functions:

Definition:

$y = \sin^{-1}x \underset{\text{same as}}{\Longleftarrow} x = \sin y$

$y = \cos^{-1}x \underset{\text{same as}}{\Longleftarrow} x = \cos y$

$y = \tan^{-1}x \underset{\text{same as}}{\Longleftarrow} x = \tan y$

Alternative Definition:

$\sin^{-1}x = \arcsin x$

$\cos^{-1}x = \arccos x$

$\tan^{-1}x = \arctan x$

Domain and Range:

Function	Domain	Range
$y = \sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1}x$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \cot^{-1}x$	$-\infty \le x \le \infty$	$0 \le y \le \pi$
$y = \sec^{-1}x$	$x \le -1, x \ge 1$	$0 \le y < \frac{\pi}{2}, \frac{\pi}{2} < y \le \pi$
$y = \csc^{-1}x$	$x \le -1, x \ge 1$	$-\frac{\pi}{2} \le y < 0, 0 < y \le \frac{\pi}{2}$

Inverse Properties:

$\sin(\sin^{-1}(x)) = x$ $\sin^{-1}(\sin(\theta)) = \theta$

$\cos(\cos^{-1}(x)) = x$ $\cos^{-1}(\cos(\theta)) = \theta$

$\tan(\tan^{-1}(x)) = x$ $\tan^{-1}(\tan(\theta)) = \theta$

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