# Graphing Conics

for Algebra students

## Conic Sections and their equations

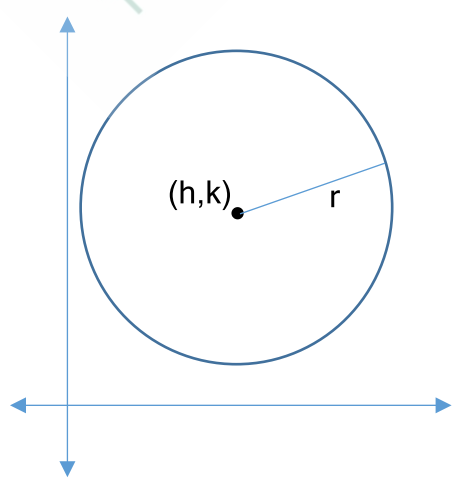
The general form of all conic equations is: $Ax^2 + Cy^2 + Cx + Ey + F = 0$

| Conic Section | Characteristic | Example |
|---|---|---|
| Circle | A = C ≠ 0 | $x^2 + y^2 - 16 = 0$ |
| Parabola | Either A=0 or C=0, but not both | $x^2 - y - 4 = 0$<br>$y^2 - x - 4y = 0$ |
| Ellipse | A ≠ C,<br>AC > 0 | $25x^2 + 16y^2 - 400 = 0$ |
| Hyperbola | A ≠ C,<br>AC < 0 | $x^2 - y^2 - 1 = 0$ |

These are the different types of conic sections in more detail:

1. Circle

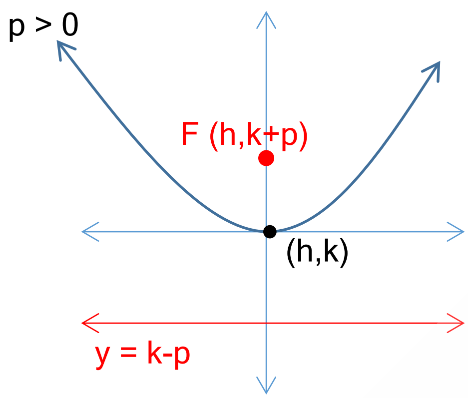
| $x^2 + y^2 = r^2$ | $(x-h)^2 + (y-k)^2 = r^2$ | $\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$ |
|---|---|---|
| • Center is (0,0)<br>• Radius is $\sqrt{r^2} = r$ | • Center is (h,k)<br>• Radius is $\sqrt{r^2} = r$ | • Center is (h,k)<br>• Radius is $\sqrt{r^2} = r$ |

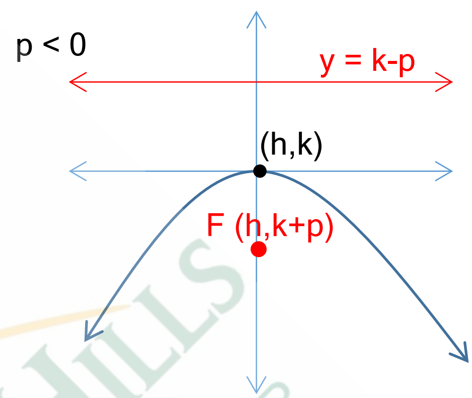


2. Parabola

a. Vertical axis of symmetry

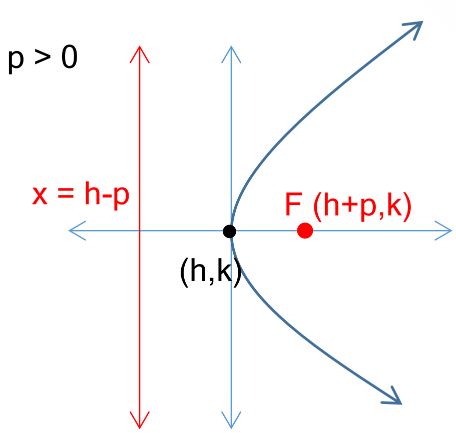
| $y-k=a(x-h)^2$ | $x^2=4py$ | $(x-h)^2=4p(y-k)$ |
|---|---|---|
| • Vertex is $(h,k)$<br>• Axis of symmetry is x = h<br>• Parabola opens up if $a>0$<br>• Parabola opens down if $a<0$ | • Vertex is (0,0)<br>• Focus is (0,p)<br>• Directrix is y = -p<br>• Vertical axis of symmetry x =0<br>• Parabola opens up if p>0<br>• Parabola opens down if p<0 | • Vertex is (h,k)<br>• Focus is (h,k+p)<br>• Directrix is y = k-p |

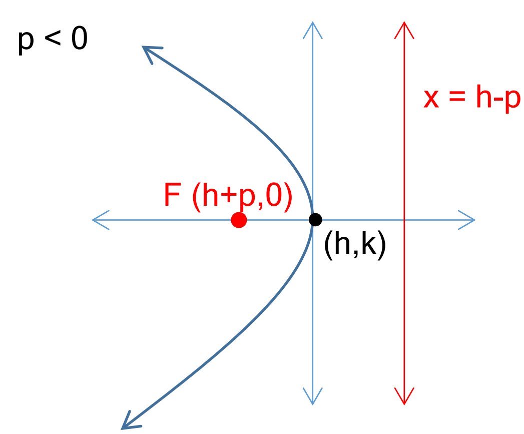



b. Horizontal axis of symmetry

| $x-h=a(y-k)^2$ | $y^2=4px$ | $(y-k)^2=4p(x-h)$ |
|---|---|---|
| • Vertex is $(h,k)$<br>• Axis of symmetry is y = k<br>• Parabola opens right if $a>0$<br>• Parabola opens left if $a<0$ | • Vertex is (0,0)<br>• Focus is (p,0)<br>• Directrix is x = -p<br>• Horizontal axis of symmetry y =0<br>• Parabola opens right if p>0<br>• Parabola opens left if p<0 | • Vertex is (h,k)<br>• Focus is (h+p,k)<br>• Directrix is x = h-p |

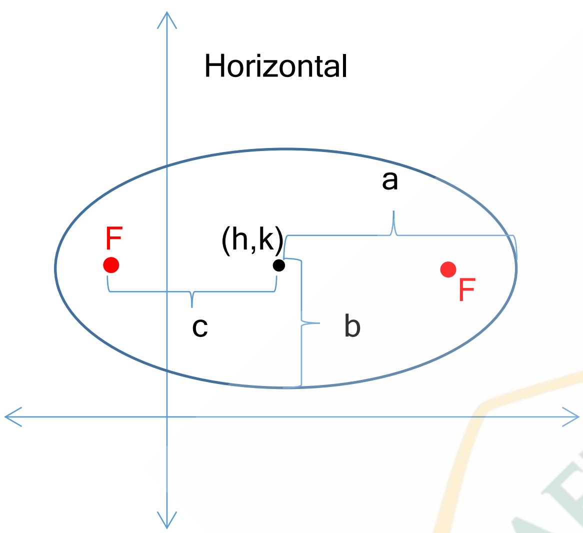


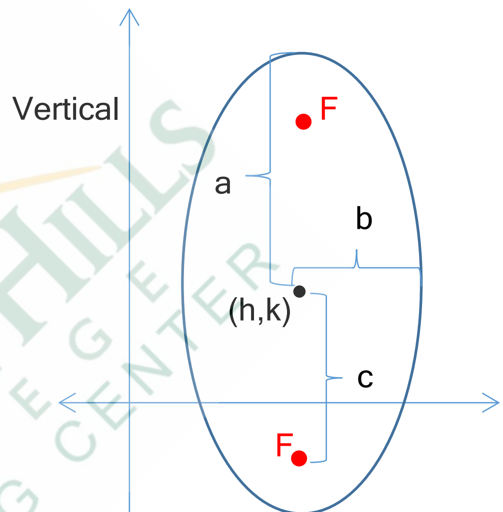



3. Ellipse, a > b > 0

    a. Major axis is x-axis (horizontal)

| $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ |
|---|---|
| • Center is (0,0)<br>• Vertices are $(\pm a, 0)$<br>• Endpoints of minor axis are $(0, \pm b)$<br>• Foci are $(\pm c, 0)$<br>• $c^2 = a^2 - b^2$ | • Center is (h,k)<br>• Vertices are $(h \pm a, k)$<br>• Endpoints of minor axis are $(0, k \pm b)$<br>• Foci are $(h \pm c, k)$<br>• $c^2 = a^2 - b^2$ |




    b. Major axis is y-axis (vertical)

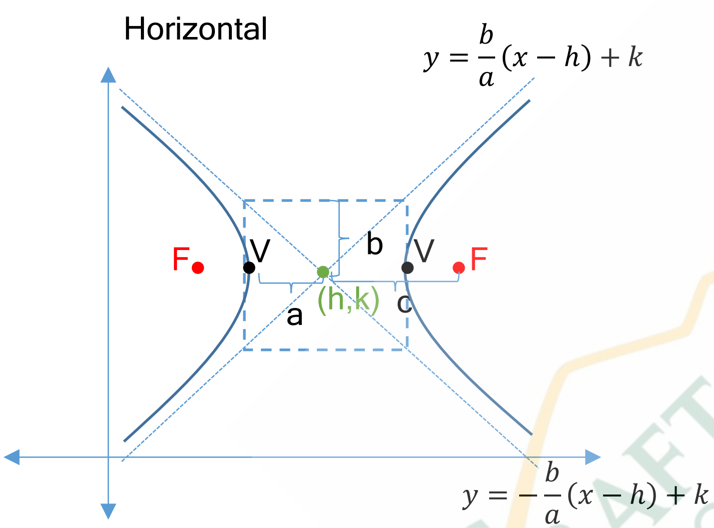
| $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ | $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ |
|---|---|
| • Center is (0,0)<br>• Vertices are $(0, \pm a)$<br>• Endpoints of minor axis are $(\pm b, 0)$<br>• Foci are $(0, \pm c)$<br>• $c^2 = a^2 - b^2$ | • Center is (h,k)<br>• Vertices are $(h, k \pm a)$<br>• Endpoints of minor axis are $(h \pm b, 0)$<br>• Foci are $(h, k \pm c)$<br>• $c^2 = a^2 - b^2$ |

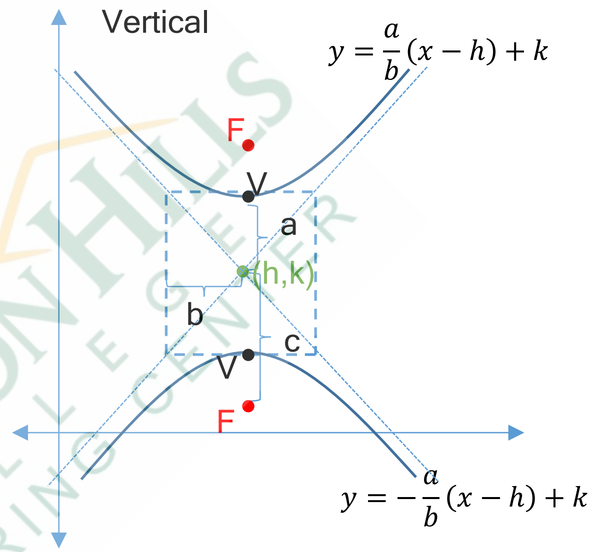
Note: The location of $a^2$, the greater value, determines whether the ellipse has a horizontal or vertical major axis.

4. Hyperbola

a. Transverse axis on x-axis (horizontal)

| $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ |
|---|---|
| • Center is (0,0)<br>• Vertices are $(\pm a, 0)$<br>• Asymptotes $y = \pm\frac{b}{a}x$<br>• Foci are $(\pm c, 0)$<br>• $c^2 = a^2 + b^2$ | • Center is (h,k)<br>• Vertices are $(h \pm a, k)$<br>• Asymptotes at $y = \pm\frac{b}{a}(x-h) + k$<br>• Foci are $(h \pm c, k)$<br>• $c^2 = a^2 + b^2$ |




b. Transverse axis on y-axis (vertical)

| $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ | $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ |
|---|---|
| • Center is (0,0)<br>• Vertices are $(0, \pm a)$<br>• Asymptotes at $y = \pm\frac{a}{b}x$<br>• Foci are $(0, \pm c)$<br>• $c^2 = a^2 + b^2$ | • Center is (h,k)<br>• Vertices are $(h, k \pm a)$<br>• Asymptotes at $y = \pm\frac{a}{b}(x-h) + k$<br>• Foci are $(h, k \pm c)$<br>• $c^2 = a^2 + b^2$ |

Note: Asymptotes for a hyperbola *always* pass through the center (h,k).