Exponential and Logarithmic Functions

for Algebra/Precalculus/Calculus Students

Rules and Properties of Exponents:

Let a and b ∈ ℝ represent bases, and let m, n, and p ∈ ℝ represent exponents, then:

1. Adding exponents
   $\boldsymbol{a^m a^n = a^{m+n}}$

   Ex: $x^7 x^2 = x^{7+2} = x^9$

2. Subtracting exponents
   $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$

   Ex: $\frac{y^6}{y^2} = y^{6-2} = y^4$

3. Multiplying exponents
   $\boldsymbol{(a^m)^n = a^{mn}}$

   Ex: $(w^{2x})^3 = w^{(2x)(3)} = w^{6x}$

4. Distributing exponents
   $\boldsymbol{(ab)^n = a^n b^n}$

   Ex: $(3y)^3 = 3^3 y^3 = 27y^3$

   $\boldsymbol{(a^m b^n)^p = a^{mp} b^{np}}$

   Ex: $(3^2 r^4)^2 = 3^{(2)(2)} r^{(4)(2)}$
   $= 3^4 r^8 = 81r^8$

   $\boldsymbol{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$

   Ex: $\left(\frac{3}{h}\right)^{2x} = \frac{3^{2x}}{h^{2x}} = \frac{9^x}{h^{2x}}$

   $\boldsymbol{\left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}}$

   Ex: $\left(\frac{q^3}{p^2}\right)^2 = \frac{q^{(3)(2)}}{p^{(2)(2)}} = \frac{q^6}{p^4}$

5. Negative exponents
   $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

   Ex: $w^{-5} = \frac{1}{w^5}$

   $\boldsymbol{\frac{1}{a^{-n}} = a^n}$

   Ex: $\frac{1}{y^{-2}} = y^2$

   $\boldsymbol{\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n}$

   Ex: $\left(\frac{y}{x}\right)^{-4} = \left(\frac{x}{y}\right)^4$

6. Zero exponents
   $\boldsymbol{a^0 = 1, a \neq 0}$

   Ex: $27^0 = 1$
   $h^0 = 1$

7. Same base
   $\boldsymbol{a^n = a^m \text{ then } n = m}$

   Ex: $2^x = 2^2 \text{ then } x = 2$

Rules and Properties of Logarithms

If b ≠ 1, M, and N ∈ $\mathbb{R}^+$, and x, y, and P ∈ $\mathbb{R}$, then:

1. $\mathbf{\log_b y = x}$ is equivalent to $\boldsymbol{b^x = y}$

   Ex: $\log_2 32 = 5$ is equivalent to $2^5 = 32$

   $\text{Log}_4\, 64 = x$ is equivalent to $4^x = 64$

2. $\mathbf{\log_b 1 = 0}$

   Ex: $\log_4 1 = 0$

   $\log_y 1 = 0$

3. $\mathbf{\log_b b = 1}$

   Ex: $\log_3 3 = 1$

   $\log_x x = 1$

4. $\mathbf{\log_e x = \ln x}$

   Ex: $\log_e 3 = \ln 3$

5. $\mathbf{\log_b b^x = x}$

   Ex: $\log_4 4^2 = 2$

   $\log_e e^u = u$

6. $\boldsymbol{b^{\log_b x} = x}$

   Ex: $10^{\log_{10} 7y} = 7y$

   $q^{\log_q 3} = 3$

7. If $\mathbf{\log_b x = \log_b y}$ *then* $\mathbf{x = y}$

   Ex: If $\log_2 z = \log_2 g$ *then* $z = g$

   If $\log 3 = \log x$ *then* $3 = x$

8. Product Rule

   $\mathbf{\log_b MN = \log_b M + \log_b N}$

   Ex: $\log_3(5k) = \log_3 5 + \log_3 k$

   $\log_e x + \log_e y = \log_e xy$

9. Quotient Rule

   $\mathbf{\log_b \frac{M}{N} = \log_b M - \log_b N}$

   Ex: $\log_{10} \frac{y}{z} = \log_{10} y - \log_{10} z$

10. Power Rule

    $\mathbf{\log_b M^P = P \cdot \log_b M}$

    Ex: $\log(xy)^2 = 2\log xy$

    $\log(5q)^{3x} = 3x\log(5q)$

    $\log(vz^{2y}) = \log v + \log z^{2y}$

    $= \log v + 2y\log z$

11. Change-of-Base Theorem

    $\mathbf{\log_b x = \frac{\log_a x}{\log_a b}}$

    Ex: $\log_5 7 = \frac{\log_{10} 7}{\log_{10} 5}$

    $\log_2 x = \frac{\ln x}{\ln 2}$