Completing the Square

For Algebra students

A trinomial that can *factor* to be a perfect square then it is called a **perfect square trinomial.**

Recall: $(x+d)^2 = (x+d)(x+d) = x^2 + 2dx + d^2$ ← Perfect Square Trinomial!

A quadratic expression has the general form: $ax^2 + bx + c$. For the above trinomial $x^2 + 2dx + d^2$, the leading coefficient is 1 $(a = 1)$, the middle coefficient is 2d $(b = 2d)$, and the constant term is d^2 $(c = d^2)$.

Relationship: The constant term c is "half of the middle term's coefficient and then squared."

$$\left(\frac{1}{2}\cdot b\right)^2 = \left(\frac{1}{2}\cdot 2d\right)^2 = (d)^2 = c$$

Ex: Find the c value that will make each trinomial a perfect square trinomial, then factor it.

(In other words…*complete the square!*)

a. $x^2 + 4x +$ ____ b. $x^2 - 10x +$ ____ c. $x^2 + 5x +$ ____

Solutions:

a. The "b"-value is 4 so we will take half of 4 and then square this entire value: $\left(\frac{1}{2}\cdot 4\right)^2 = (2)^2 = 4$.
The perfect square trinomial is $x^2 + 4x + \boxed{4}$ and factoring this trinomial we get: $x^2 + 4x + 4 = (x+2)^2$.

b. The "b"-value is -10 so we will take half of -10 and then square this entire value: $\left(\frac{1}{2}\cdot -10\right)^2 = (-5)^2 = 25$.
The perfect square trinomial is $x^2 - 10x + \boxed{25}$ and factoring this trinomial we get: $x^2 - 10x + 25 = (x-5)^2$.

c. The "b"-value is 5 so we will take half of 5 and then square this entire value: $\left(\frac{1}{2}\cdot 5\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$.
The perfect square trinomial is $x^2 + 5x + \boxed{\frac{25}{4}}$ and factoring this trinomial we get: $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$.

A COMPLETED SQUARE TRINOMIAL FACTORS IN A CERTAIN WAY: $x^2 + bx + \left(\frac{1}{2}\cdot b\right)^2 = \left(x + \left(\frac{1}{2}\cdot b\right)\right)^2$

Completing the square is often used to solve quadratic equations.

Solving Equations:

Ex. Solve the equation by completing the square: $x^2 + 6x + 2 = 0$

Step 1: $x^2 + 6x \quad = -2$

Step 2: $x^2 + 6x + \_\_ = -2 + \_\_\_\_$ (leading coefficient is already 1)

Step 3: $x^2 + 6x + \mathbf{9} = -2 + \mathbf{9}$ $\left(\frac{1}{2} \cdot b\right)^2 = \left(\frac{1}{2} \cdot 6\right)^2 = (3)^2 = 9$

Step 4: $(x + 3)^2 = 7$

Step 5: $x + 3 = \pm\sqrt{7}$

Step 6: $x = -3 \pm \sqrt{7}$

$x = -3 + \sqrt{7}$

$x = -3 - \sqrt{7}$

This is the **completing the square** part!

Ex. Solve the equation by completing the square: $3x^2 - 15x - 21 = 0$

Step 1: $3x^2 - 15x \quad = 21$

Step 2: $x^2 - 5x + \_\_ = 7 + \_\_\_\_$ (divide every term by 3)

Step 3: $x^2 + 5x + \mathbf{\frac{25}{4}} = -2 + \frac{25}{4}$ $\left(\frac{1}{2} \cdot b\right)^2 = \left(\frac{1}{2} \cdot 5\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

Step 4: $\left(x + \frac{5}{2}\right)^2 = \frac{17}{4}$

Step 5: $x + \frac{5}{2} = \pm\sqrt{\frac{17}{4}}$

Step 6: $x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$

$$x = -\frac{5}{2} + \frac{\sqrt{17}}{2}$$

$$x = -\frac{5}{2} - \frac{\sqrt{17}}{2}$$

Steps for Solving an Equation by Completing the Square

Step 1: Move the constant "c" to the other side of the equation—away from terms with variables.

Step 2: If the leading coefficient is something other than 1, then divide EVERYTHING by that number.

Step 3: Take half of the "b"-value (divide by 2) and then square it. Add that new number to both sides of the equation. Simplify the right-hand side of equation.

Step 4: Shrink (factor) the left side of the equation to $\left(x + \frac{b}{2}\right)^2$

Step 5: Square root both sides to get rid of the square. Remember to put a $\pm$ on the right hand side.

Step 6: Get x by itself. Simplify your solution and we are done! Simplify, if possible!