CRAFTON HILLS COLLEGE TUTORING CENTER

Conditions for Series Convergence and Divergence

Calculus Students

Depending on the form of the series, the following rules can be used to test special series for convergence or divergence. *

Series/Test	Form of Series	Condition for convergence	Condition for divergence	Comments
Geometric Series	$\sum_{k=0}^{\infty} ar^k$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	If $\lvert r\rvert < 1$, then $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$
Divergence Test	$\sum_{k=1}^{\infty} a_k$	Not Applicable	$\lim_{k\to\infty} a_k \neq 0$	Cannot be used to prove convergence
Integral Test	$\sum_{k=1}^{\infty} a_k$ *where* $a_k = f(k)$ and f *is continuous, positive, and decreasing*	$\int_1^{\infty} f(x)dx < \infty$	$\int_1^{\infty} f(x)dx = \infty$	The value of the integral is **not** the value of the series
p-series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p > 1$	$p \leq 1$	Can often relate other series to this one (Comparison Tests)
Ratio Test	$\sum_{k=1}^{\infty} a_k$ *where* $a_k > 0$	$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} < 1$	$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} > 1$	Inconclusive if the limit = 1
Root Test	$\sum_{k=1}^{\infty} a_k$ *where* $a_k \geq 0$	$\lim_{k\to\infty} \sqrt[k]{a_k} < 1$	$\lim_{k\to\infty} \sqrt[k]{a_k} > 1$	Inconclusive if the limit = 1
Comparison Test	$\sum_{k=1}^{\infty} a_k$ *where* $a_k > 0$	$0 < a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ *converges*	$0 < b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ *diverges*	Must generate the series for b_k
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ *where* $a_k > 0, b_k > 0$	$0 \leq \lim_{k\to\infty} \frac{a_k}{b_k} < \infty$ and $\sum_{k=1}^{\infty} b_k$ *converges*	$\lim_{k\to\infty} \frac{a_k}{b_k} > 0$ and $\sum_{k=1}^{\infty} b_k$ *diverges*	Must generate the series for b_k
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^k a_k$, *where* $a_k > 0$ and $0 < a_{k+1} \leq a_k$	$\lim_{k\to\infty} a_k = 0$	$\lim_{k\to\infty} a_k \neq 0$	The remainder: $\lvert R_n\rvert < a_{n+1}$
Absolute Convergence	$\sum_{k=1}^{\infty} a_k$	$\sum_{k=1}^{\infty} \lvert a_k\rvert$ *converges*		Applies to arbitrary series

*Adapted from: Briggs, Cochran, *Calculus: Early Transcendentals* 2nd Edition

Updated: April 2017